Heavy Flavor Spectroscopy on the Lattice

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Outline

- Introduction
- Control of Systematic Errors
- Summary of Lattice Approaches
- Results
  - Onium
  - Flavorful Heavy Mesons
  - Heavy baryons
- Outlook

Introduction

Try to address these questions:

- Which spectroscopic quantities can lattice address well?
- Which quantities are difficult for lattice calculations?
- How can an outsider get a sense of the quality of a lattice result?
- Which approaches are currently pursued?
- What are some results?
- What are expectations for future improvements?
  - or at least, from which groups can we expect results?

Easy vs. Hard

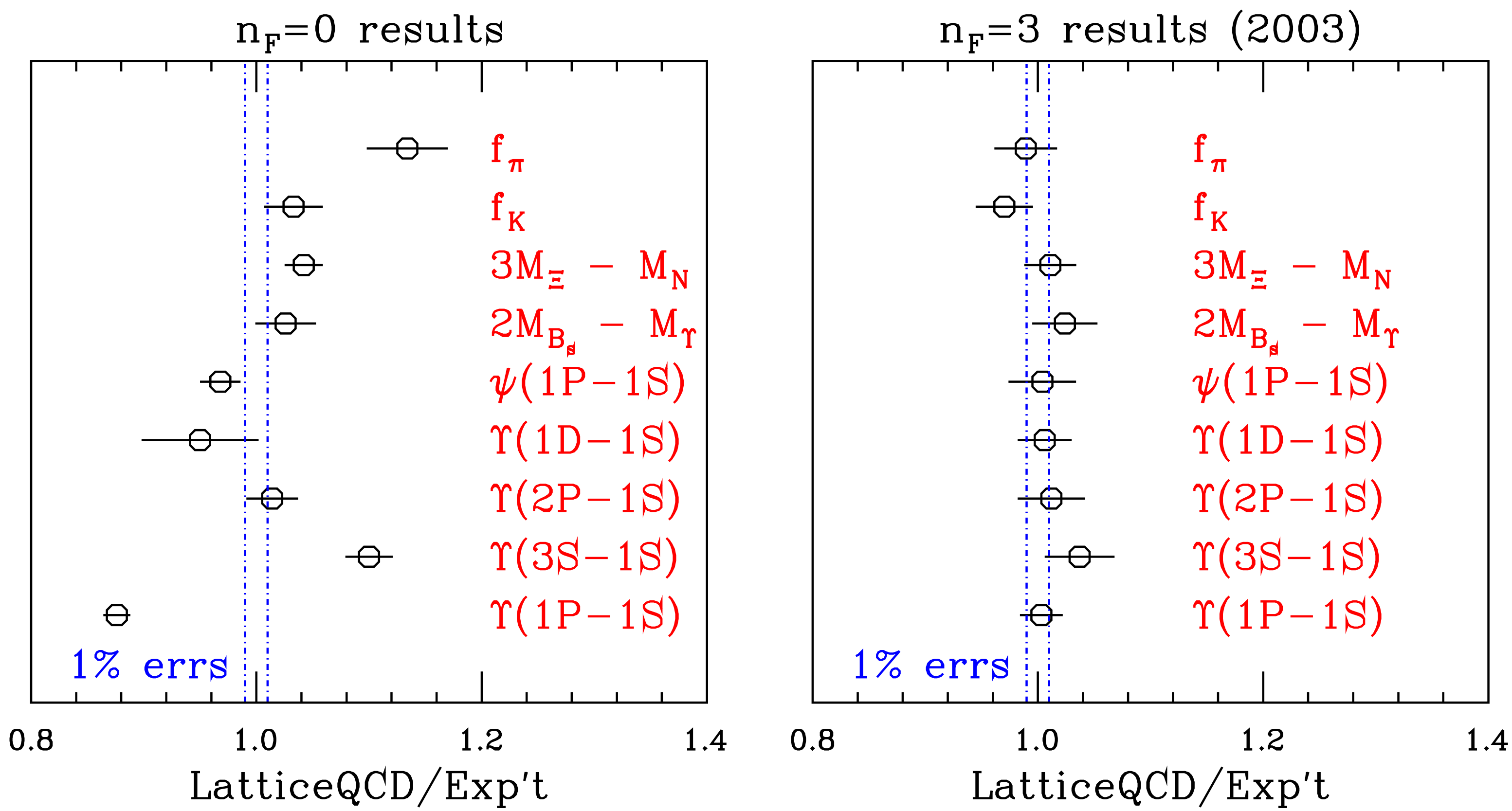
- Excited states are much harder than ‘ground’ states.
- Lightest state with given quantum numbers is easiest.
- If two states have the same quark content, but different spin or angular momentum, lighter state will have smaller errors.
- States that decay strongly, or are just below threshold are very challenging.
- Calculations are easier when ma is small, but there are techniques for dealing with heavy quarks.

Quality Control

How can an outsider get a sense of the quality of a lattice result?

- Lattice errors come from both statistics and systematics
- Statistical errors are usually easy to judge. However,
  - beware of fits and extrapolations that do not properly account for correlations and autocorrelations,
  - beware of calculations with unreasonable or missing confidence levels of fits.
- Dynamical quark content is important. It can result in errors that cannot be controlled or estimated.
- $N_f = 0$, 2, $2+1$?

Importance of Dynamical Quarks



FNAL, HPQCD, MILC and UKQCD [PRL 92, 0022001 (2004)]

Control of Systematic Errors

To carry out a simulation we must select certain physical parameters:

- lattice spacing (a) or gauge coupling (β)
- grid size ($N_s^3 \times N_t$)
- sea quark masses ($m_{u,d}$, m_s) (m_c ?)

To control systematic error we must

- take continuum limit
- take infinite volume limit
- extrapolate to light quark mass; can work at physical s quark mass

- The best calculations have a range of lattice spacing and extrapolate to $a = 0$.
- Typical lattice spacings are about 0.15 to 0.05 fm.
- Few groups run on multiple volumes and extrapolate. Most use a single box size. In some cases, a finite volume correction is applied, but this has not been done for heavy quark systems.
- Light hadrons probably require a box size of $>$2.5 fm depending upon the lightest pion mass, but is it possible that heavy spectroscopy does not require such large boxes.
- Extrapolation for light quark mass can be very important, but some states like Υ, J/ψ, D_s, B_s have no light valence quark. That should make chiral extrapolation easier. (Only sea quarks need extrapolation.)

Summary of Lattice Approaches

- Various groups use different formulations of the quark and gauge actions.
- They should agree in the continuum limit, but have different systematic errors at nonzero a.
- The number of approaches has been proliferating and makes for interesting competition.
- New approaches are numerically more complicated, but their errors should have a higher power of a, and approach continuum limit sooner.
- Also, anisotropy in space and time lattice spacings can be introduced.

Glossary

- Gauge Action
  - Wilson gauge action (plaquette)
  - Symanzik improved
  - Iwasaki
  - DBW2 (doubly blocked Wilson 2)
  - Perfect
- Quark Action
  - Wilson
  - Kogut-Susskind (or staggered)
  - Clover (including Stout, FLIC, Twisted Mass)
  - Domain Wall
  - Overlap
  - asqtad
  - HISQ

Glossary II

- Heavy Quark Action
  - Any of the above
  - Clover with Fermilab interpretation
  - NRQCD (nonrelativistic QCD)
  - Static approximation (infinitely heavy quark)

Results

- This is a fairly active field. I checked Lattice 2005, 2006 and 2007 for papers:
  - Lattice 2005: 6 papers
  - Lattice 2006: 13 papers
  - Lattice 2007: 10 papers
- Obviously cannot go through all the results, so I will show the titles and select a few calculations.

Lattice 2005

- Pseudoscalar mass and decay constant in Lattice QCD with Exact Chiral Symmetry, PoS(LAT2005)041, T.W. Chiu, T.H. Hsieh, J.Y. Lee, P.H. Liu and H.J. Chang [quenched, overlap; one lattice spacing]
- Charmonium spectroscopy on dynamical anisotropic lattices, PoS(LAT2005)029, K.J. Juge, A.O Cais, M.B. Oktay, M.J. Peardon and S.M. Ryan [$N_f = 2$, anisotropic, stout link fattening]
- Charmonium Spectrum with all-to-all propagators, PoS(LAT2005)086, K.J. Juge, A.O Cais, M.B. Oktay, M.J. Peardon and S.M. Ryan [see above]
- Heavy-light mesons with domain wall fermions, PoS(LAT2005)096, S. Ohta, H.W. Lin and N. Yamada [quenched, DBW2, one lattice spacing]
- NRQCD results on the MILC extra coarse ensemble, PoS(LAT2005)217, I. Allison, C.T.H. Davies and A. Gray [$N_f = 2 + 1$, asqtad, NRQCD for b, one lattice spacing]
- B_s meson excited states from the lattice, PoS(LAT2005)205, A.M. Green, J. Ignatius, M. Jahma, J. Koponen, C. McNeile and C. Michael [$N_f = 2$, clover, static, one lattice spacing]

Lattice 2006

- Heavy flavor physics from lattice QCD, PoS(LAT2006)017, T. Onogi [review, not much on spectrum]
- A study of the B_s–$\bar{B}_s$ mass and width difference in 2+1 flavor lattice QCD, PoS(LAT2006)081, R. Evans [asqtad, one lattice spacing]
- Charm physics with highly improved staggered quarks, PoS(LAT2006)082, E. Follana [$N_f = 2 + 1$, asqtad, HISQ, one lattice spacing]
- $\bar{\Lambda}$, λ_1, and m_b in three-flavor lattice QCD, PoS(LAT2006)083, E. Freeland [$N_f = 2 + 1$, asqtad, clover, three lattice spacings]
- M_b and f_{B_s} from a combination of HQET and QCD, PoS(LAT2006)084, D. Guazzini [quenched, clover, three lattice spacings]
- The spectrum of charmonium-like vector mesons in lattice QCD, PoS(LAT2006)170, T.W. Chiu [quenched, optimal domain wall, one lattice spacing]
- Update on onium masses with three flavors of dynamical quarks, PoS(LAT2006)175, S. Gottlieb [$N_f = 2 + 1$, asqtad, clover, four lattice spacings]
- B_s and B_c mesons in lattice QCD with exact chiral symmetry, PoS(LAT2006)180, T.H. Hsieh [quenched, optimal domain wall, one lattice spacing]

Lattice 2006, continued

- Charmed spectroscopy from a nonperturbatively determined relativistic heavy quark action in full QCD, PoS(LAT2006)184, H.W. Lin [$N_f = 2 + 1$, domain wall, one lattice spacing]
- Hybrid charmonium from lattice QCD, PoS(LAT2006)186, X.Q. Luo [quenched, anisotropic, three lattice spacings]
- Exotic and higher spin mesons in charmonium region, PoS(LAT2006)187, N. Mathur [quenched, anisotropic, clover, one lattice spacing]
- Heavy baryon mass spectrum from lattice QCD with 2+1 flavors, PoS(LAT2006)191, H. Na [$N_f = 2 + 1$, asqtad, only charm, one lattice spacing]
- Spectrum of radial, orbital and gluonic excitations of charmonium, PoS(LAT2006)193, M.B. Oktay [$N_f = 2$, anisotropic, one lattice spacing]
- Orbital and radial excitations of static-light mesons, PoS(LAT2006)196, S.M. Ryan [$N_f = 2$, anisotropic, one lattice spacing, will study 2nd volume]
- Exploratory study of the D_s spectrum in 2+1 domain wall QCD with heavy overlap, PoS(LAT2006)202, A. Trivini [one lattice spacing]

Lattice 2007

- B meson excitations with chirally improved light quarks, PoS(LATTICE 2007)091, T. Burch [quenched and dynamical, static heavy quarks, two quenched, one dynamical lattice spacings]
- Charmonium spectrum including higher spin and exotic states, PoS(LATTICE 2007)094, C. Ehmann and G. Bali [$N_f = 2$, clover, one lattice spacing]
- The B-meson mass splitting from non-perturbative quenched lattice QCD, PoS(LATTICE 2007)100, D. Guazzini [quenched, operator renormalization]
- Dirac b quark on the lattice, PoS(LATTICE 2007)105, T.W. Chiu, T.H. Hsieh, C.H. Huang and K. Ogawa [quenched, optimal domain wall, one lattice spacing]
- P- and D-wave spin-orbit splittings in heavy-light measons, PoS(LATTICE 2007)112, J. Koponen [$N_f = 2$, clover, static heavy quarks]
- Effects of the disconnected flavor singlet corrections on the hyperfine splitting in charmonium, PoS(LATTICE 2007)116, L. Levkova [$N_f = 2 + 1$, asqtad, one lattice spacing]
- Charm spectroscopy on dynamical 2+1 flavor domain wall fermion lattices with a relativistic heavy quark action, PoS(LATTICE 2007)117, M. Li and H.W. Lin [one lattice spacing]

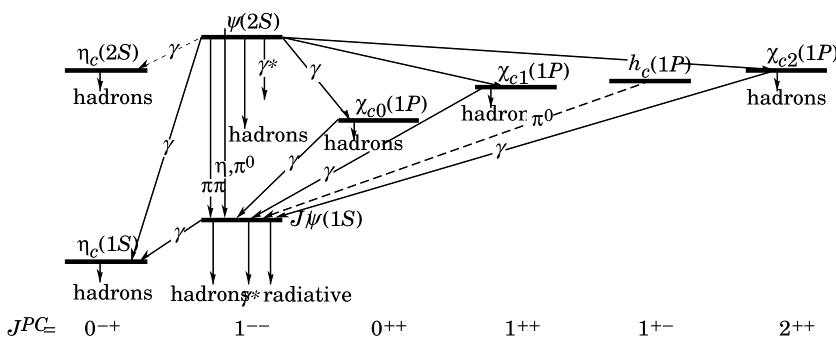
Lattice 2007, continued

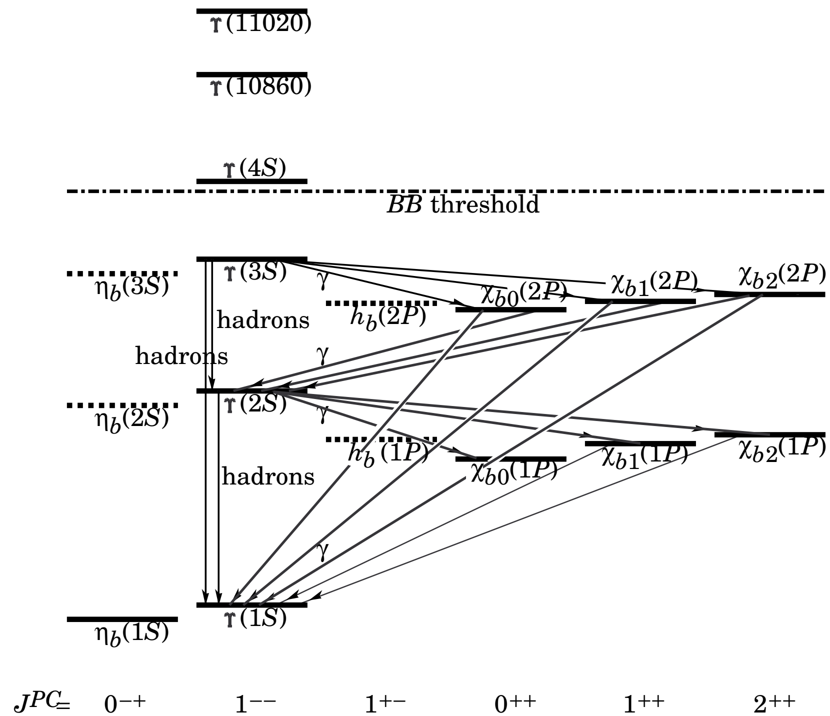
- Charm and bottom heavy baryon mass spectrum from lattice QCD with 2+1 flavors, PoS(LATTICE 2007)124, H. Na and S. Gottlieb [$N_f = 2+1$, asqtad, one lattice spacing]
- D_s spectrum from $N_f = 2$ anisotropic lattices, PoS(LATTICE 2007)128, M.B. Oktay [stout link fattening, one lattice spacing]
- High precision determination of the D and D_s decay constants and masses from lattice QCD, PoS(LATTICE 2007)353, E. Follana, C.T.H. Davies, P. Lepage and J. Shigemitsu [$N_f = 2+1$, asqtad, HISQ heavy quarks, three lattice spacings]

Heavy Quark Spectrum

- Best tests of LGT are states that require no chiral extrapolation, are stable to strong decay (and far from threshold).
  Examples: J/Ψ, Υ, D_s, B_s
- FNAL Lattice/MILC collaborations have used Clover quarks to study charmonium
  [PoS LAT2005: 203(2006); hep-lat/0510072]
- HPQCD/UKQCD collaborations have used NRQCD accurate to order v^4 to study bottomonium on MILC configurations
  [PRD 72, 09407 (2005); hep-lat/0507013]
- RBC is using a relativistic heavy quark action with $N_f = 2 + 1$ domain wall configurations [PoS(LAT2006)184, (Lattice 2007)117]

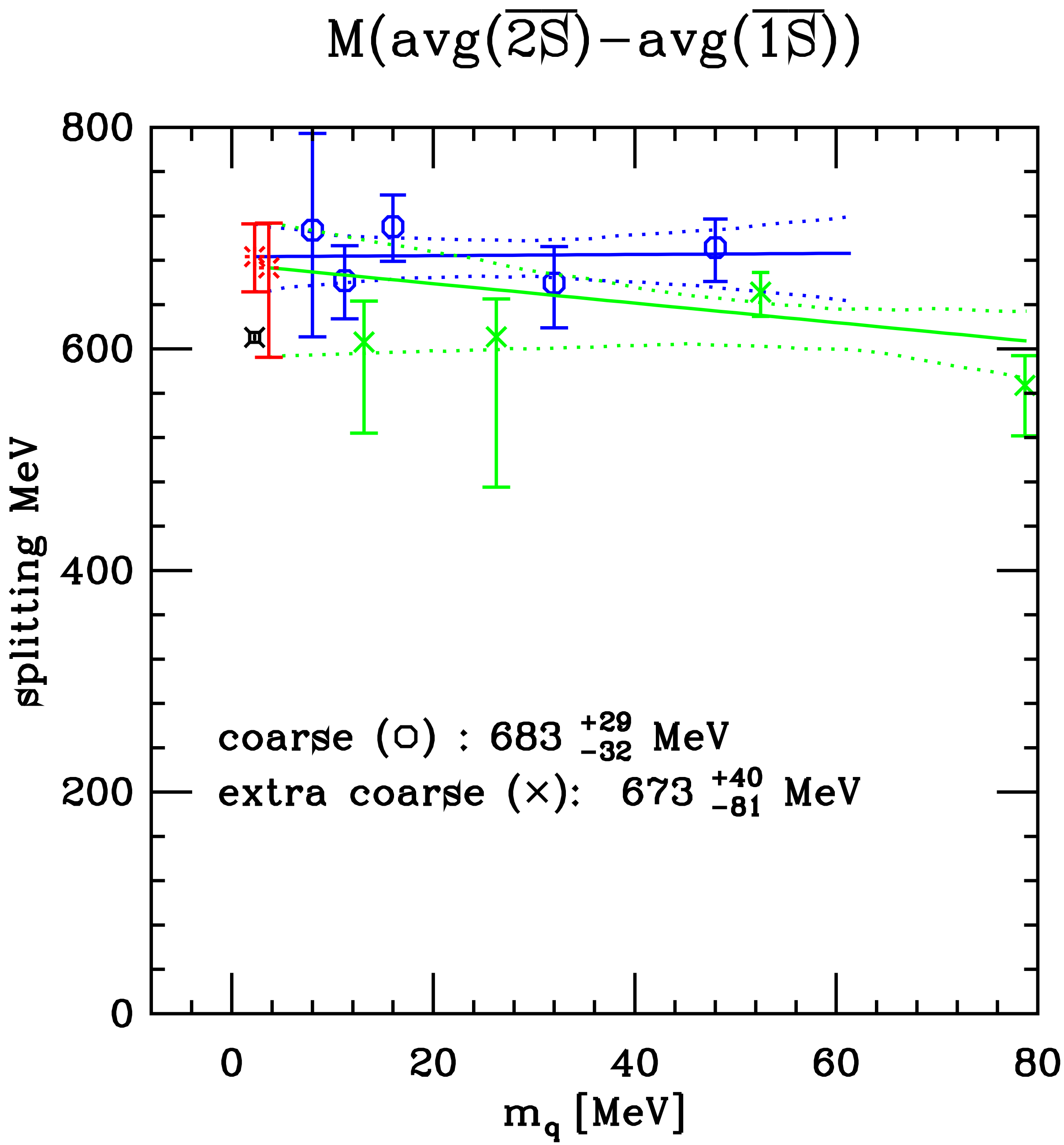
Observed Spectrum



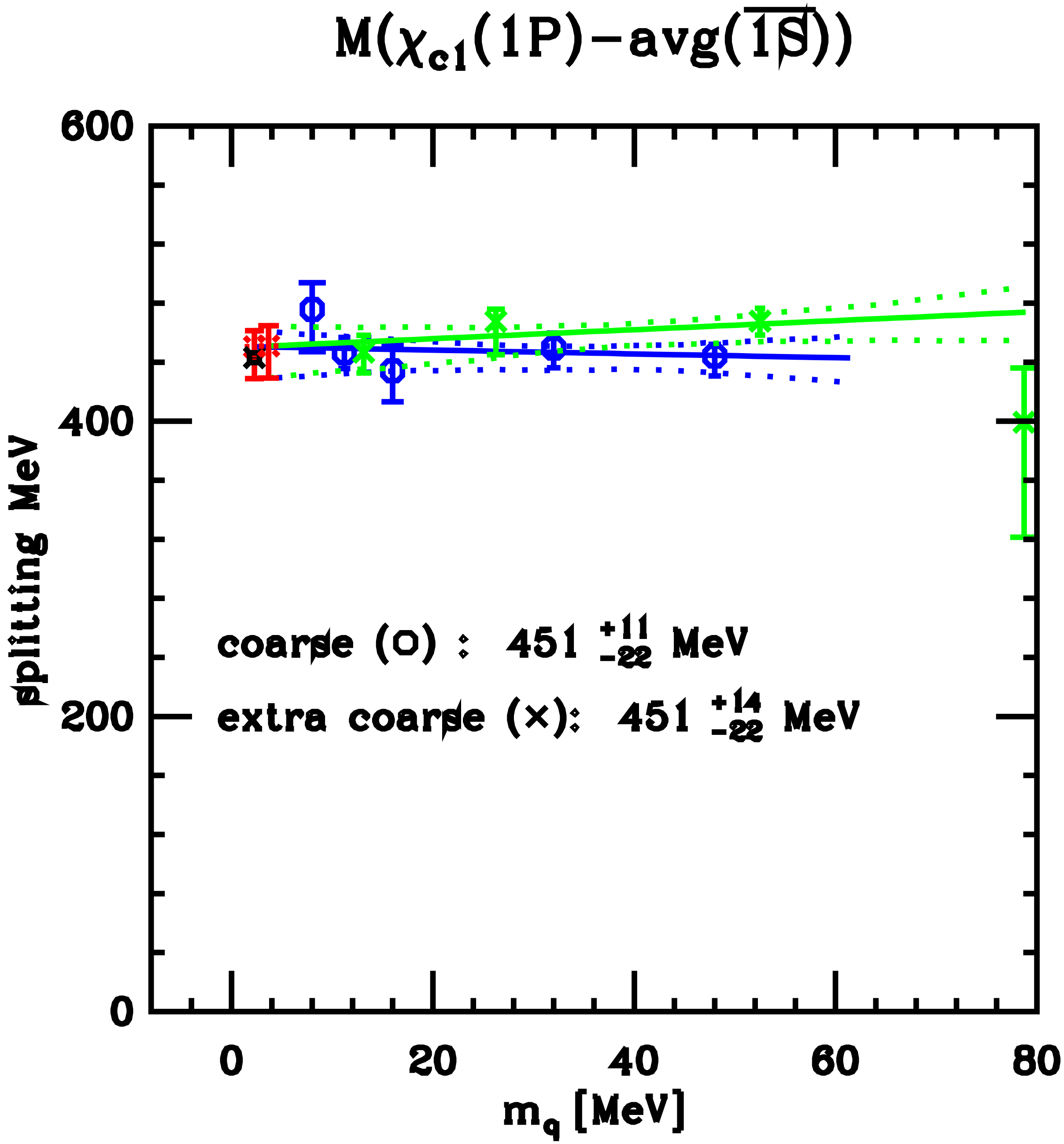


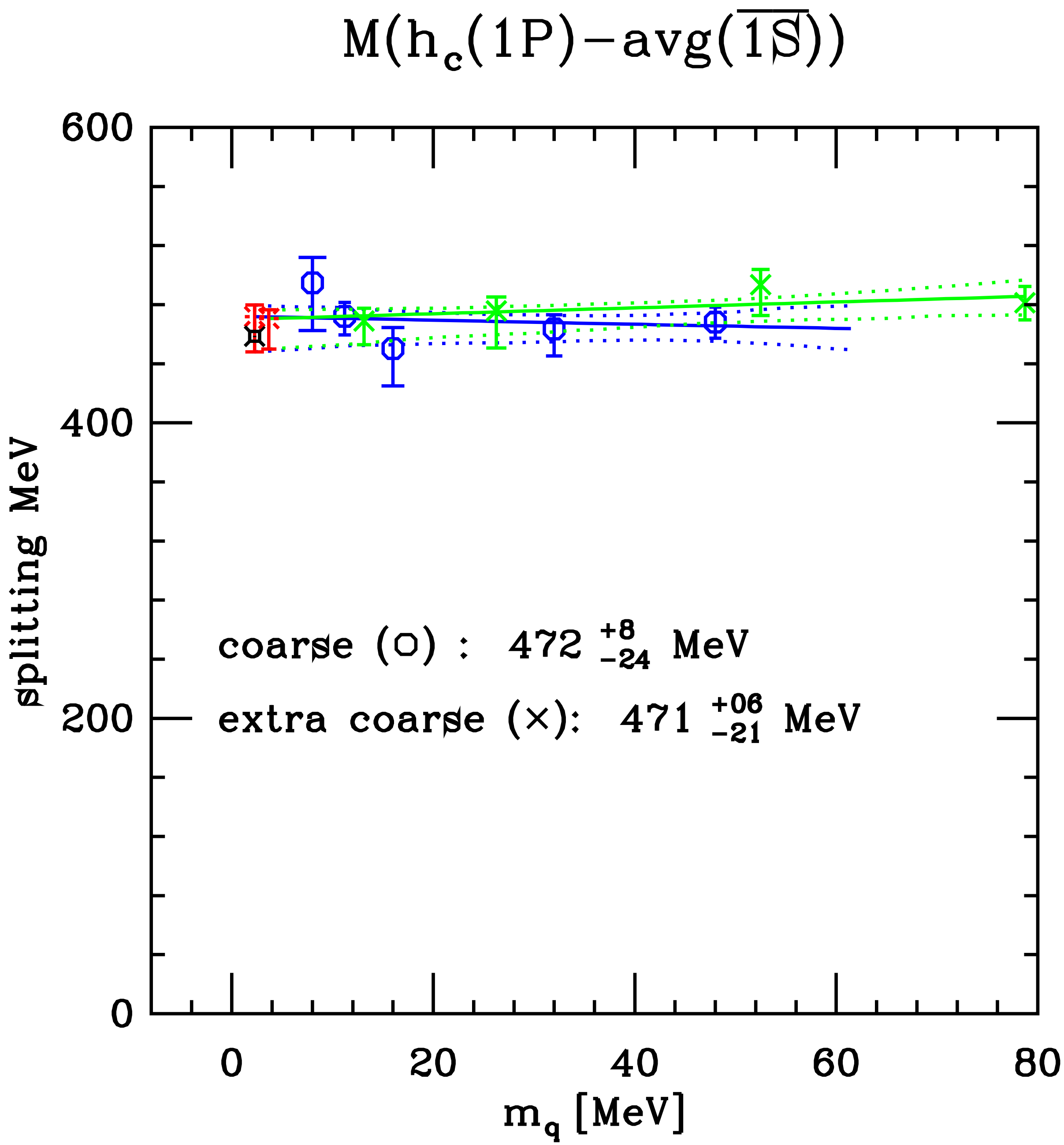
[S. Eidelman et al., PL B592, 1 (2004)]

Charmonium w. Clover

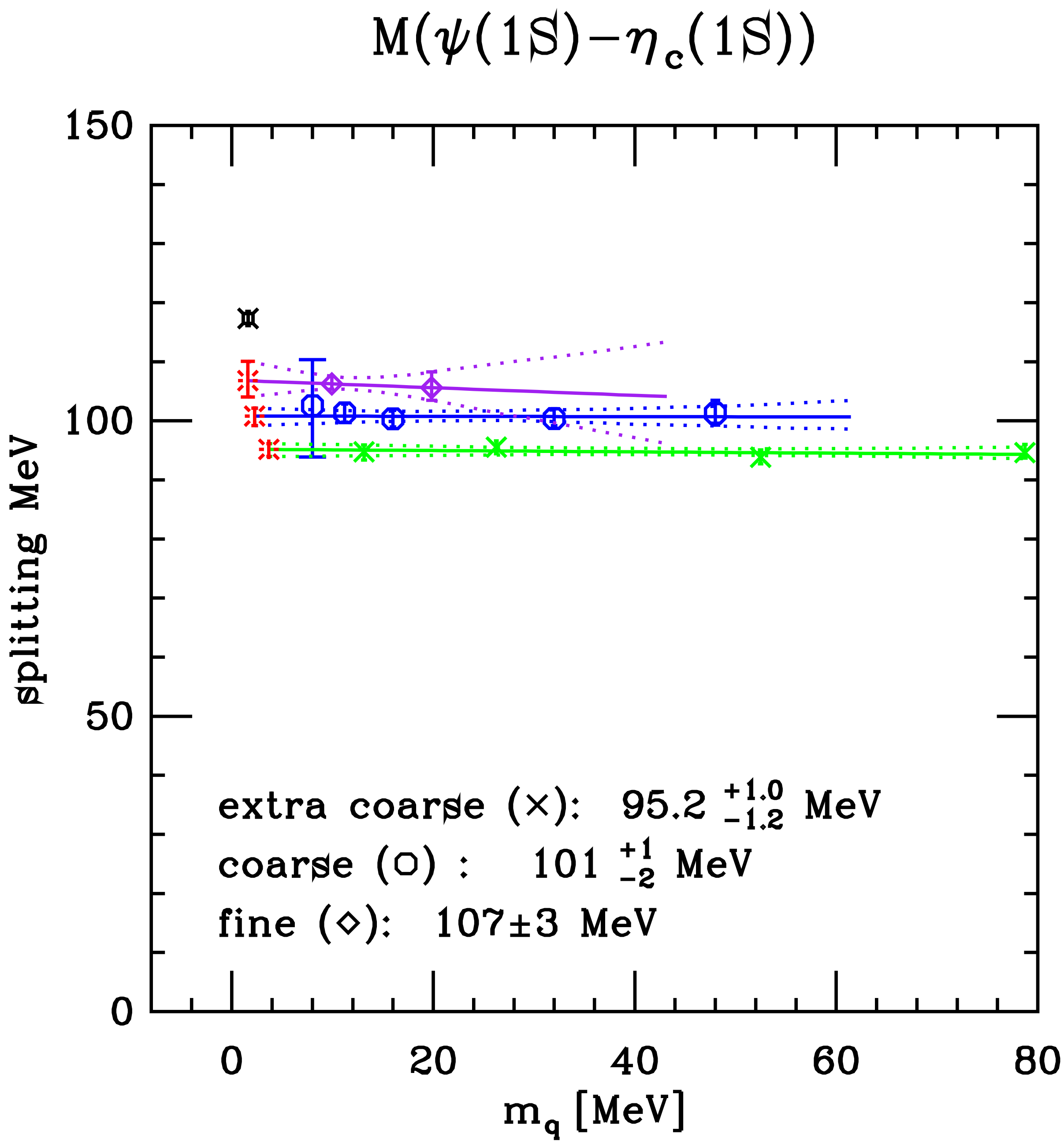


Fine Structure

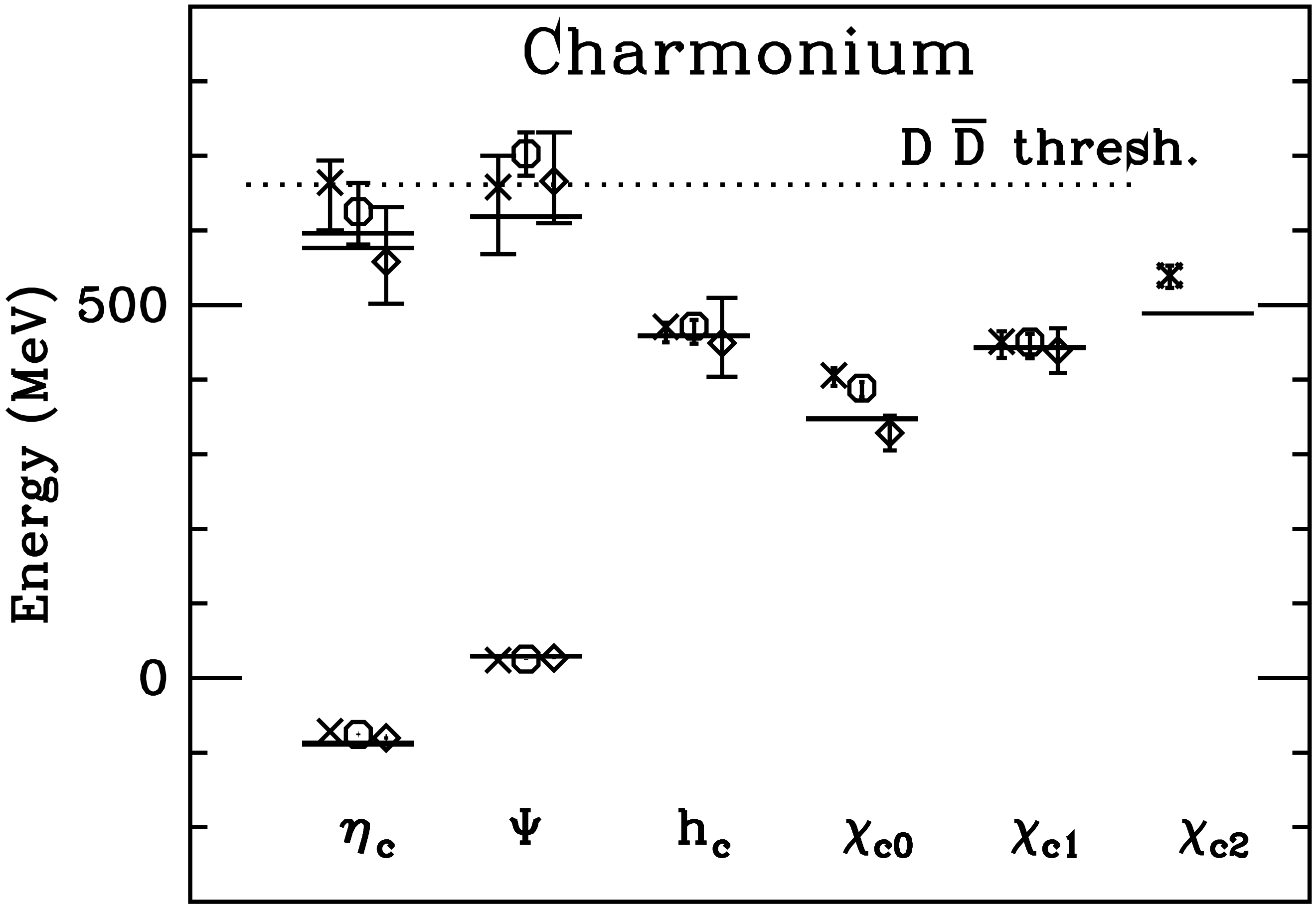



M(h_c(1P)−avg(1S))
600
400
200
0
splitting MeV
coarse (○) : 472 +8 −24 MeV
extra coarse (×): 471 +06 −21 MeV
0
20
40
60
80
m_q [MeV]

1S Hyperfine Splitting

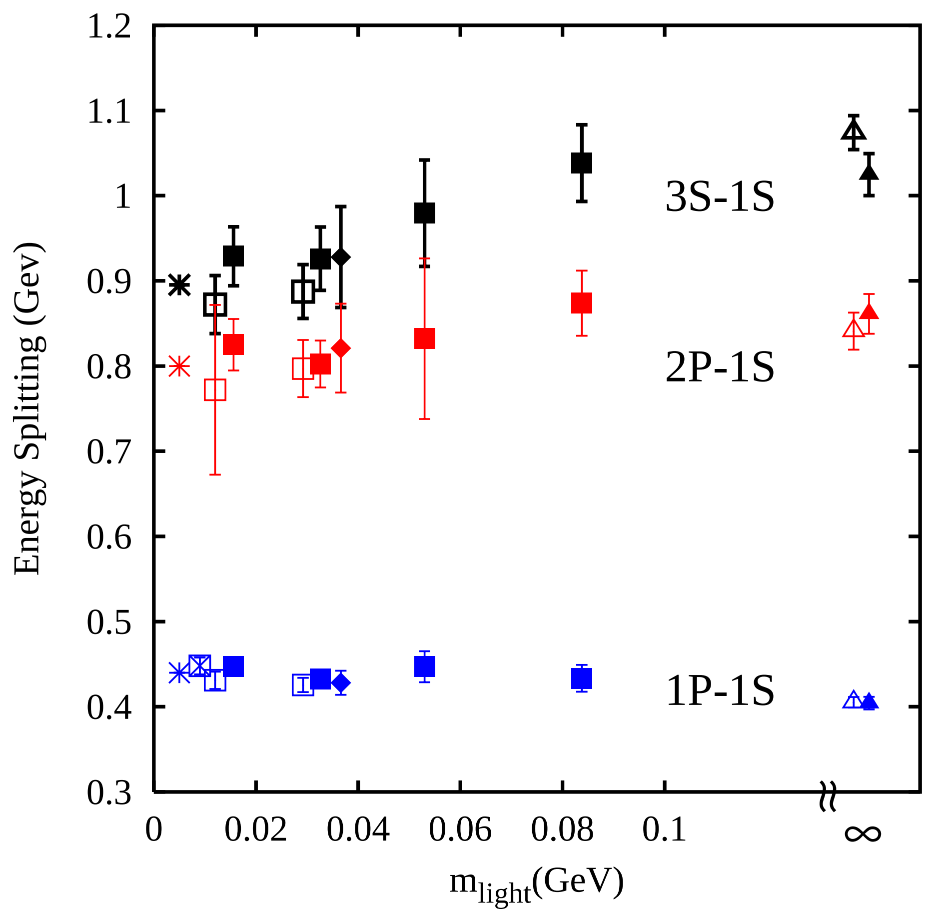


Summary of Charmonium Spectrum



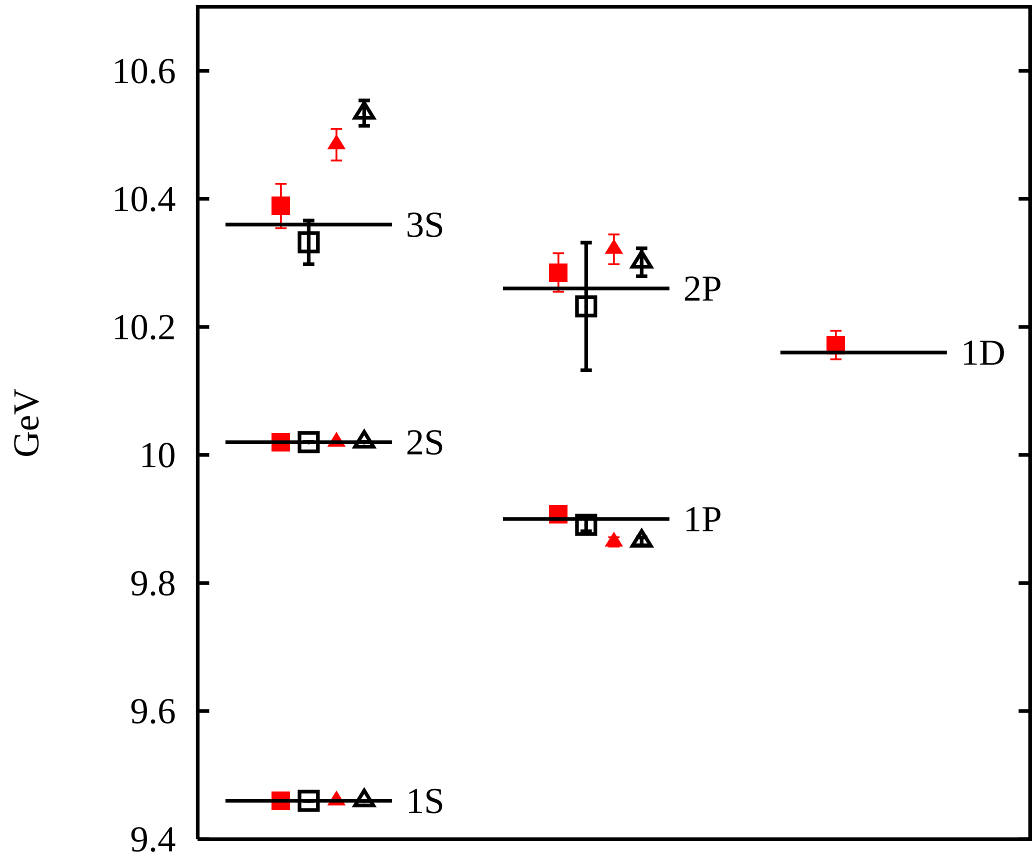
Results for supercoarse, coarse and fine lattices based on linear chiral extrapolation. chi_{c2} has only been studied on one coarse ensemble.

Onium splitting w. NRQCD



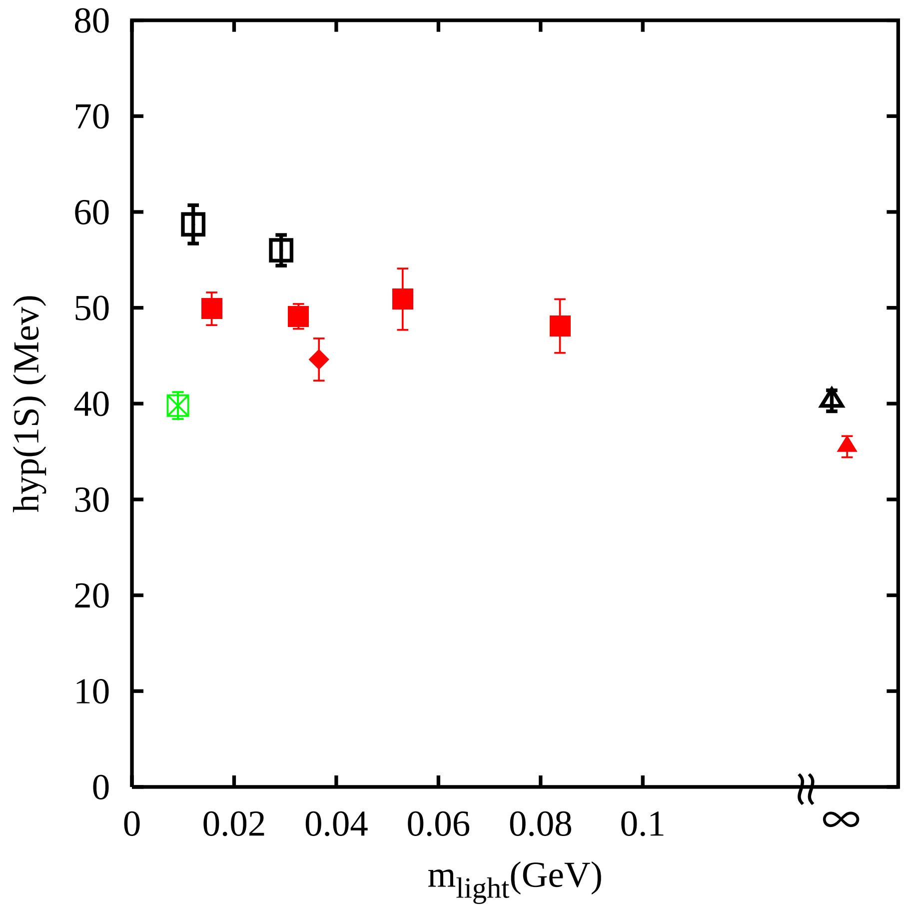
Spin-independent splittings in the Υ spectrum. The S state is always the $^3S_1$. The P state from the lattice calculations is the $^1P_1$; from experiment it is the spin-average of $^3P_{0,1,2}$. Closed squares (coarse) open squares (fine), crossed square (supercoarse). Closed diamonds are from the coarse $n_f = 2$ run. Closed and open triangles are from quenched coarse and fine ensembles respectively. Bursts give the experimental results.

Fine, hyperfine structure



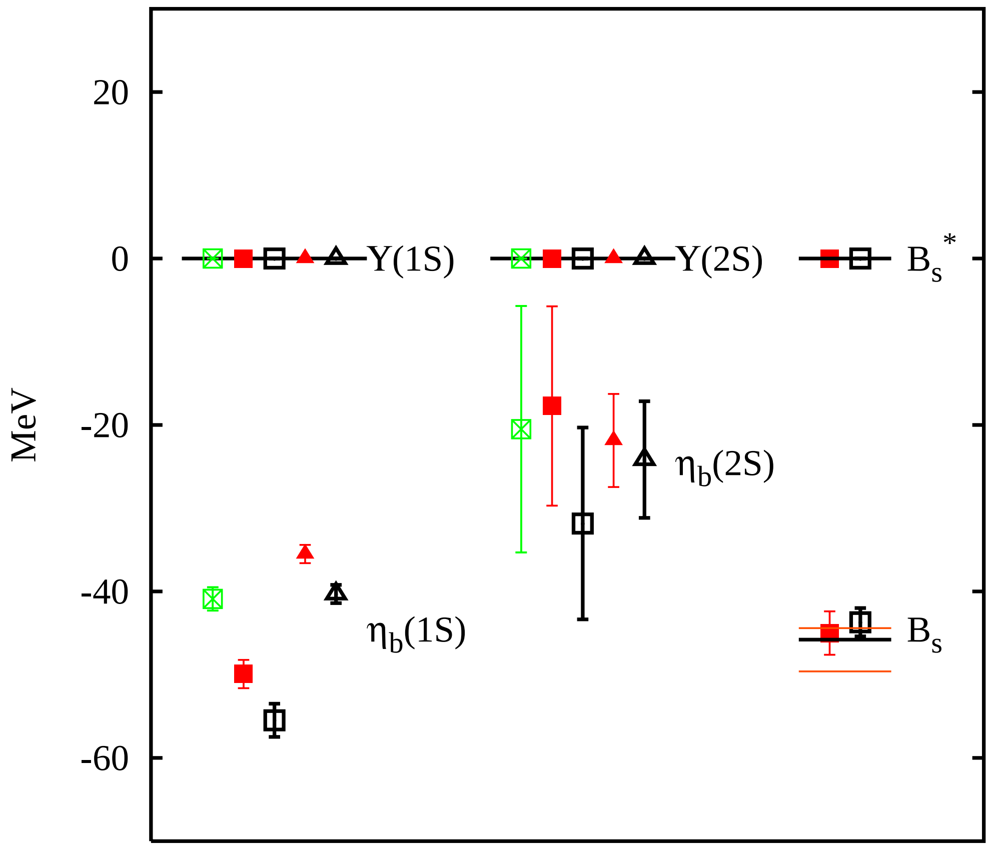
The Υ spectrum of radial and orbital levels, taking the most chiral points from previous page. Closed and open symbols are from coarse and fine lattices respectively. Squares are dynamical configurations and triangles are quenched. Lines represent experiment.

1S Hyperfine splitting



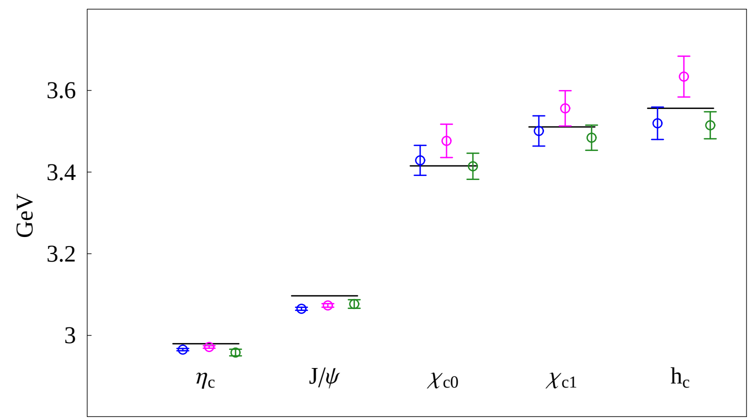
Ground-state hyperfine splitting in the Υ system as a function of the dynamical light quark mass. Symbols as in previous graphs. Errors include both statistical/fitting errors from determining the splitting and from determining the lattice spacing scale.

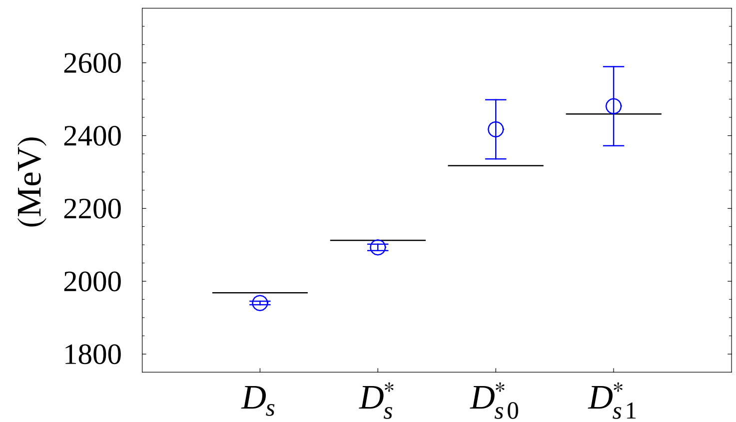
Hyperfine splitting



Hyperfine splittings in the Υ and B_s systems. Additional systematic errors are 25% and discussed in the text. The faint lines give the experimental limits on the $B_s^* - B_s$ splitting and the dark line gives the experimental result for $B^* - B$ (PDG) .

Onium and Heavy-Light with DWF



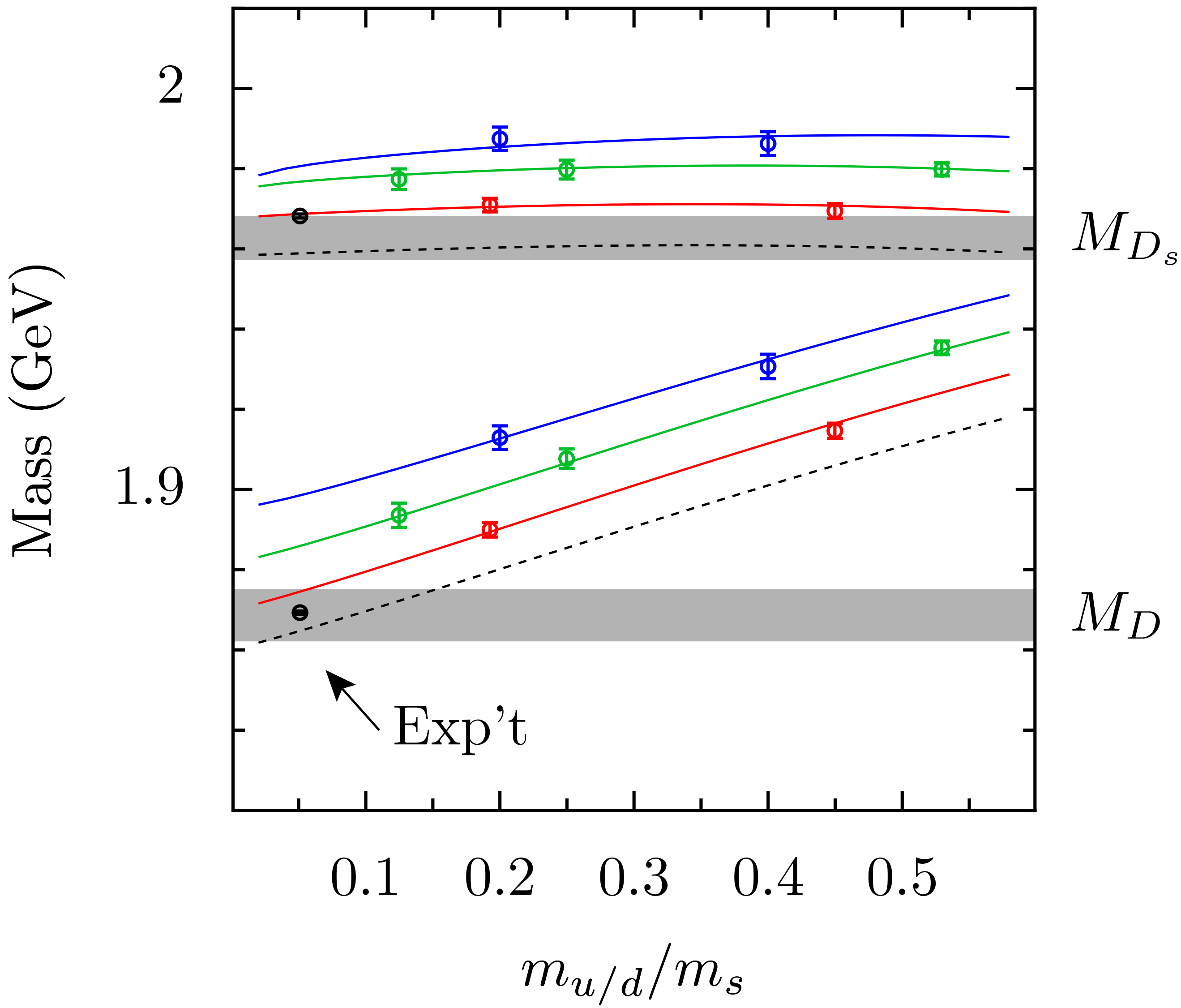


H.-W. Lin/RBC [PoS(LAT2006)184]

$M_{D^*} - m_D = 154(16)$ MeV 142

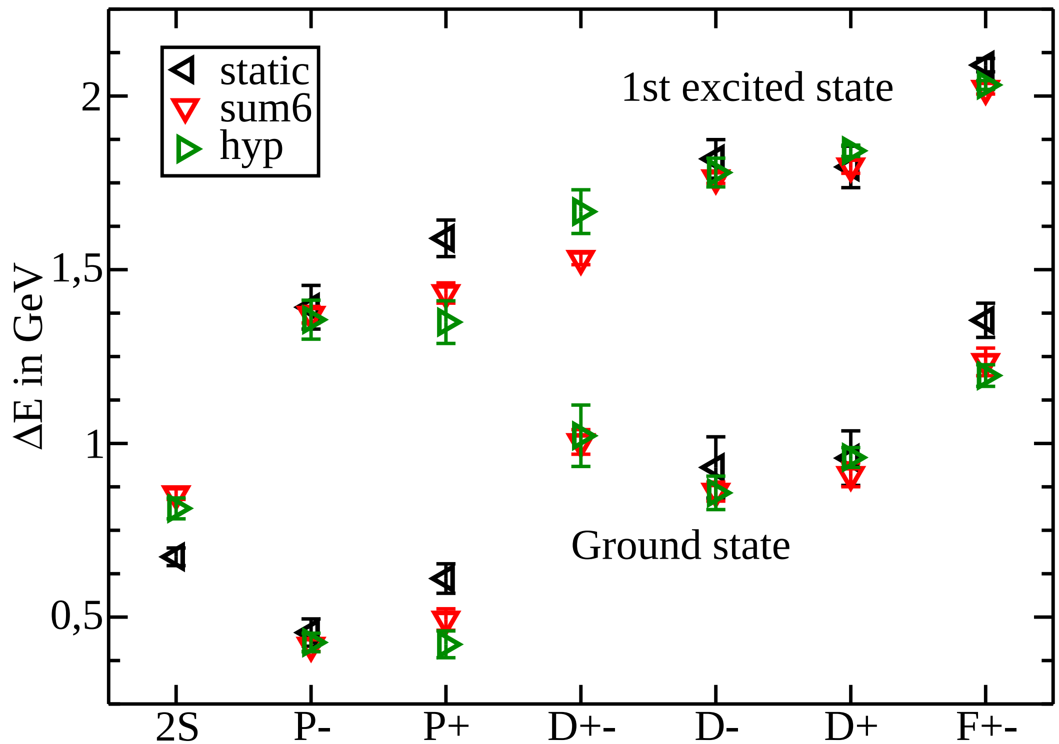
$M_{D_s^*} - m_{D_s} = 153(7)$ MeV 144

Heavy-light mesons



Follana, Davies, Lepage, Shigemitsu, PoS(Lattice 2007)353: HISQ charm on asqtad configurations. $M_{D_s} = 1.963(5)$ GeV (vs. 1.968), $M_D = 1.869(6)$ GeV(vs. 1.869).

Static calculation

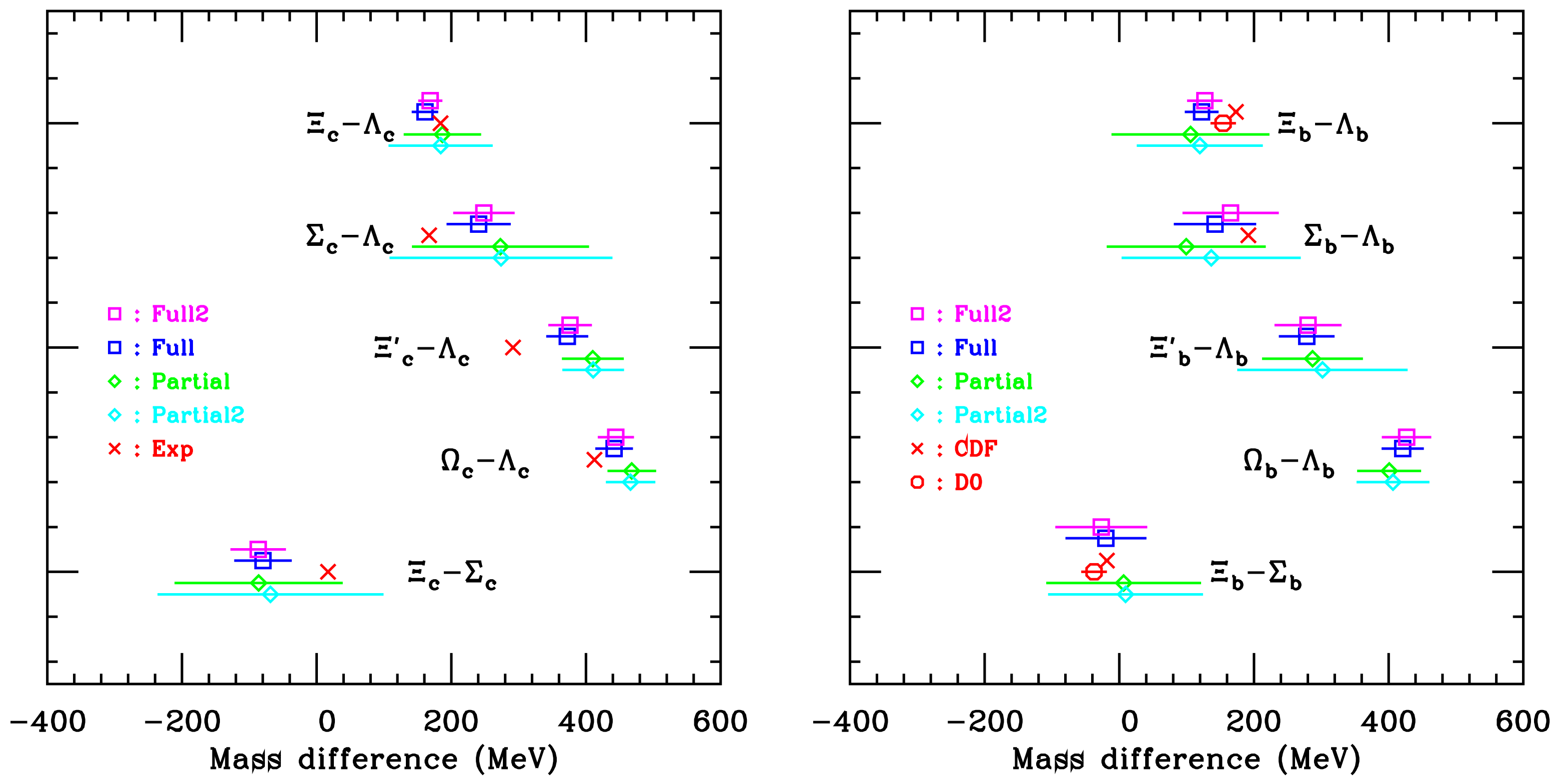


Koponen: PoS(Lattice 2007) 112: Energy spectrum of the heavy-light meson. Here L$+$($-$) means that the light quark spin couples to angular momentum L giving the total $j =$ L $\pm 1/2$. The 2S is the first radially excited L$= 0$ state. The D$+-$ is a mixture of the D$-$ and D$+$ states, and likewise for the F$+-$. Energies are given with respect to the S-wave ground state (1S). Here $a = 0.110(6)$ fm is used to convert the energies to physical units. The error bars

Heavy Baryons

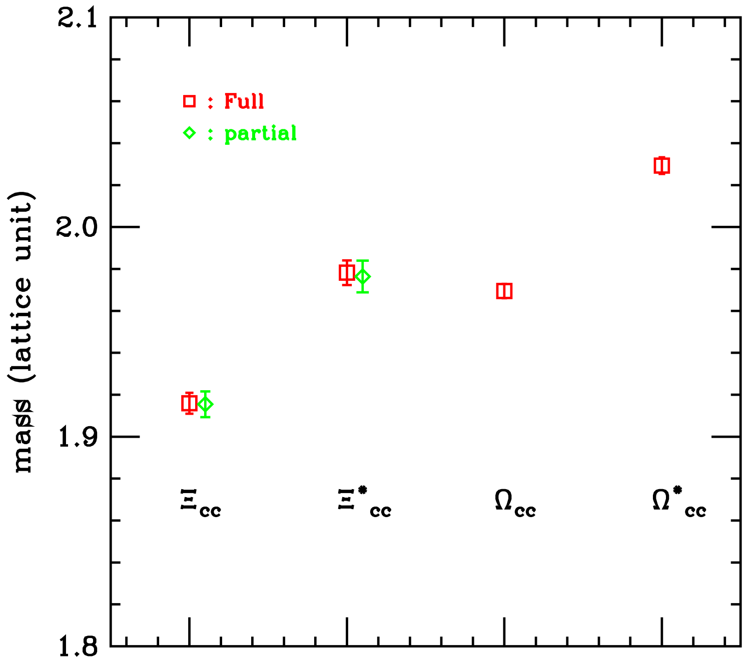
- Study of heavy baryons has a long history starting with UKQCD [PRD 54 (1996) 3619].
- Four or five other groups have studying heavy baryons, but all within the quenehced approximation until 2006.
- My graduate student Heechang Na began his study using the MILC $N_f = 2 + 1$ asqtad lattices with $a = 0.12$ fm. This is currently being extended to two different lattice sizes.
- We also need to improve our chiral extrapolation to make better use of all mass combinations available.
- Some preliminary results follow. Current splitting errors roughly 25–60 MeV.

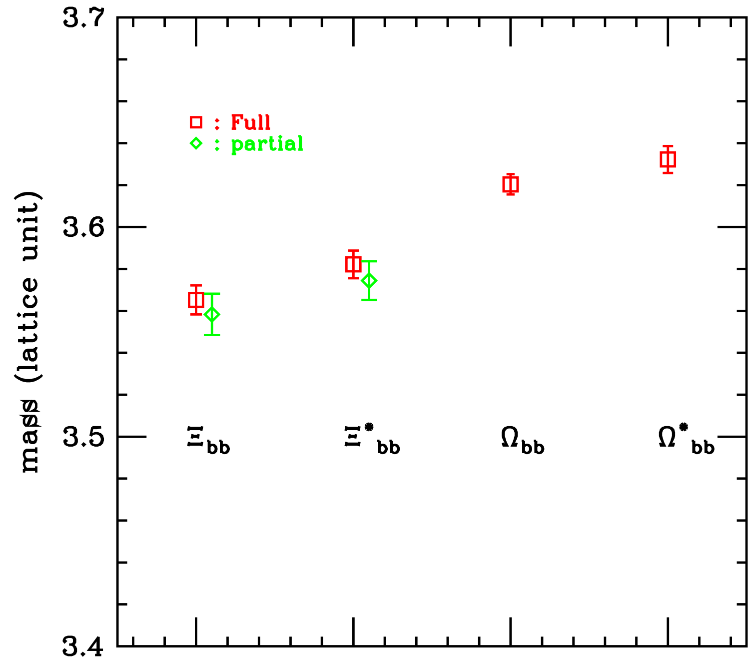
Singly Heavy Baryon Splittings



[PoS(LATTICE 2007)124]

Doubly Heavy Baryon Splittings





[PoS(LATTICE 2007)124]

Outlook

- Quite a few groups are interested in heavy quark spectroscopy.
- Only a couple of groups have so far used lattices with a realistic dynamical quark content.
- Only with asqtad have several lattice spacings been studied.
- This should change in the near future, as several groups are pursuing large scale lattice generation projects.

- Through the ILDG, many ensembles will be shared with the international community.
  - ETMC: twisted mass QCD, $N_f = 2+1+1$, $a = 0.09$, 0.07 fm
  - QCDSF: stout link, clover, $N_f = 2+1$, $a = 0.08$ fm
  - UKQCD/RBC: domain wall, $N_f = 2+1$, $a = 0.11$, 0.09 fm
  - CP-PACS/JLQCD: clover/Iwasaki, $N_f = 2+1$, $a = 0.12$, 0.10, 0.07 fm
  - PACS-CS: clover/Iwasaki, $N_f = 2+1$, larger volumes
  - JLQCD overlap/Iwasaki, $N_f = 2$, $a = 0.12$, then $N_f = 2+1$
  - LHPC: anisotropic, stout smeared clover, $N_f = 2$, $2+1$, $a = 0.1$, 0.125 fm